

FREQUENCY SLOPE ESTIMATION AND ITS APPLICATION FOR NON-STATIONARY SINUSOIDAL PARAMETER ESTIMATION

Axel Röbel

IRCAM-CNRS-STMS, Analysis-Synthesis Team
Paris, France
roebel@ircam.fr

ABSTRACT

In the following paper we investigate into the estimation of sinusoidal parameters for sinusoids with linear AM/FM modulation. It will be shown that for linear amplitude and frequency modulation only the frequency modulation creates additional estimation bias for the standard sinusoidal parameter estimator. Then an enhanced algorithm for frequency domain demodulation of spectral peaks is proposed that can be used to obtain an approximate maximum likelihood estimate of the frequency slope, and an estimate of the amplitude, phase and frequency parameter with significantly reduced bias. An experimental evaluation compares the new estimation scheme with previously existing methods. It shows that significant bias reduction is achieved for a large range of slopes and zero padding factors. A real world example demonstrates that the enhanced bias reduction algorithm can achieve a reduction of the residual energy of up to 9dB.

1. INTRODUCTION

Additive (or sinusoidal) models are often used for the representation, analysis or transformation of music or speech signals [1, 2]. An important step that is necessary to obtain the sinusoidal model consists of the estimation of the amplitude, frequency and phase of the sinusoids from the peaks of the discrete Fourier transform. The estimation is rather simple as long as the signal is stationary. A standard method for this estimation is the quadratically interpolated FFT (QIFFT) [3]. The QIFFT estimator uses the bin at the maximum of each spectral peak together with its two neighbors to establish a 2nd order polynomial model of the log amplitude and unwrapped phase of the peak. The amplitude and frequency estimates of the sinusoid that is related to the spectral peak are then derived from the height and frequency position of the maximum of the polynomial. The evaluation of the phase polynomial at the frequency position provides the estimate of the phase of the sinusoid.

For non-stationary sinusoids the parameter estimation becomes more difficult because the QIFFT algorithm is severely biased whenever the frequency is not constant. The term bias refers to the systematic estimation error. It describes the offset of the estimator that exists even if no measurement noise is present. For the partials in natural vibrato signals the estimation bias of the QIFFT estimator accounts for a significant amount of residual energy. It is the major reason for the perceived voiced energy in the residual of vibrato signals. A number of algorithms with low estimation bias for non stationary sinusoids have been proposed. Algorithms that try to implement a MLE are generally assuming that the amplitude of the sinusoids are constant. As example we refer to an algorithm that is based on signal demodulation employing an initial search over a grid of frequencies and frequency slopes and

a final fine-tuning of the parameters using an iterative maximization of the amplitude of the demodulated signal [4]. Similar as for multi-component signals with stationary sinusoids the MLE of sinusoidal parameters for multi-component signals with FM modulated sinusoids is rather costly as in this case a highly nonlinear and high dimensional cost function needs to be maximized [5]. Due to the computational savings and despite the fact that windowing reduces the estimator efficiency the windowing technique is generally preferred if the signal contains more than a single sinusoid. The algorithms that employ analysis windows for the parameter analysis of AM/FM modulated sinusoids generally rely on the fact that the analysis window is approximately Gaussian such that a mathematical investigation becomes tractable [6, 7, 3]. The method presented in [3] is special in that it tries to extend its range to other analysis windows by means of a set of linear bias correction functions. The resulting estimator is computationally rather efficient and achieves small bias for standard windows as long as the zero padding factor is sufficiently large (≥ 3) and the frequency chirp rate is relatively small.

In the following paper we present a bias correction scheme for sinusoidal parameter estimation of sinusoids with linear AM/FM modulation. As a first step we provide a mathematical foundation for the conjecture that linear amplitude modulation does not create any additional bias for the QIFFT estimator. With respect to bias reduction we may therefore ignore the amplitude modulation of the signal. Then we extend an initial version of our bias reduction method that has been proposed originally in [10]. The basic ideas of the algorithm are similar to [4] in that the algorithm is based on signal demodulation and maximization of the amplitude of the demodulated signal to find the sinusoidal parameters. In contrast to [4] however, the algorithm allows the use of an analysis window and does not use time domain demodulation. Therefore, it can be applied if the signal contains more than a single sinusoid. Moreover, the initial 2-dimensional grid search of the algorithm presented in [4] is avoided due to the fact that first, a simple and efficient initial estimate of the frequency slope estimate is used, and second, the frequency and frequency slope estimation have been decoupled. After demodulating the frequency slope the standard QIFFT estimator can be applied to obtain an estimate of the sinusoidal parameters. Due to the fact that the QIFFT estimator has small bias for constant frequency sinusoids the resulting estimate is significantly improved. In [10] it has been shown that demodulation can be achieved by means of spectral deconvolution using only the peak to be analyzed and a properly selected and scaled demodulation kernel. In the original version the frequency slope estimate was entirely handled by the frequency slope estimator in [3].

The version to be presented here is a refined version of the original demodulation algorithm. The enhancements include a

new procedure to improve the initial estimate of the frequency slope reducing the remaining bias for large frequency slopes. Furthermore, the constraint to use the same analysis window for the signal spectrum and the demodulation kernel has been removed. Accordingly, it becomes possible to trade-off bias against noise sensitivity. A computationally efficient version of the algorithm using precomputed and linearly interpolated demodulation kernels is presented. We describe an experimental comparison of the new frequency slope estimator with the previous version and the approach presented recently in [3] and an experimental evaluation of different bias reduction schemes for a real world vibrato signal.

The organization of the article is as follows. In section 2 we will show how the bias of the standard estimators is related to the frequency slope. In section 3 we will describe the demodulation scheme and the improved frequency slope estimator. In section 4 we present experimental results for the frequency slope estimation algorithm as well as for the bias reduction scheme by means of comparing the results of different algorithms. Furthermore we compare different bias reduction methods by means of comparing the residual energy of the sinusoidal model of a real world vibrato signal. In section 5 we conclude with an outlook on further improvements.

2. ESTIMATION BIAS

The signal model that will be used in the following assumes a linear evolution for amplitude and frequency trajectories. Accordingly, a complex discrete time sinusoid can be represented as

$$s(n) = (A + an) \exp(i(\phi + 2\pi\omega_0 n + \pi D n^2)). \quad (1)$$

Here A is the mean amplitude of the signal and a is the amplitude slope. ϕ is the phase of the sinusoid at time $n = 0$, ω_0 is its mean frequency and D is the frequency slope. Note, that all frequency values are normalized with respect to the samplerate. The center of the analysis window is located at time 0 such that an ideal estimator should provide (A, ω_0, ϕ) as estimates for amplitude, frequency, and phase. The model equation (1) is necessarily time limited due to the fact that we assume $A + an > 0$ for all sample positions n that are used in a signal analysis.

As introduction into the problem we will summarize the sources of bias that are known to exist for the standard QIFFT estimator and discuss their implications in the context of parameter estimation for linear AM/FM modulated sinusoids.

First, there is the use of a second order model for interpolating the spectral bins. While this is systematically wrong for the present sinusoidal model, it does not have any direct relation to the fact that the sinusoidal parameters are varying. Because the QIFFT algorithm will be used extensively, it is nevertheless important to reduce this type of bias as far as possible. This can be achieved by means of zero padding the analysis window or, as demonstrated recently, by means of simple bias correction functions [8].

Second, there is the cross component bias that is due to other sinusoidal components. The technique that is generally used to reduce this bias is windowing. The analysis window reduces the sidelobes of the sinusoidal components such that the cross component bias of distant sinusoidal components can be effectively reduced. Note however, that the reduction of the sidelobe amplitudes is always accompanied by an increased mainlobe width. Therefore, the windowing technique will slightly increase the cross component bias for nearby components. Moreover, due to the tapering

of the signal at the frame borders the noise sensitivity of the parameter estimation is slightly increased. In the following we will assume that the sinusoidal components are resolved such that the frequency distance between two sinusoids is always larger than the width of the mainlobe of both components. In this case the cross component bias will stay nearly the same for stationary and non-stationary components such that the cross component bias will only change marginally with the modulation of the sinusoids.

Third, there is the bias due to the non-stationary parameters. For the sinusoidal model in equation (1) and a Gaussian analysis window the bias has been analyzed mathematically in [7]. The result shows, that the QIFFT algorithm suffers from additional bias due to parameter variation only if the frequency slope $D \neq 0$. In this case, the estimation of all three basic parameters are biased and the bias increases with the absolute value of D .

To study the dependency of the estimation bias on the frequency slope for arbitrary analysis windows we split the sinusoidal model in equation (1) into two parts, a sinusoid with constant amplitude A and sinusoid with mean amplitude 0 and amplitude slope a . Then we investigate the properties of the spectra of the individual parts and use the linearity of the Fourier transform to draw conclusions for the complete spectrum. We first write the DFT of the signal equation (1) using analysis window $W(n)$ as follows

$$S(\omega) = \sum_{n=-\infty}^{\infty} W(n)(A + an)e^{i(\phi + 2\pi\omega_0 n + \pi D n^2)} e^{-i2\pi\omega n}. \quad (2)$$

Assuming the analysis window to be even symmetric we can make use of the symmetry relations and remove all parts of the sum in equation (2) that are odd symmetric in n . As a result equation (2) simplifies into

$$S(\omega) = S_c(\omega) + S_l(\omega) \quad \text{with} \quad (3)$$

$$S_c(\omega) = Ae^{i\phi} \sum_{n=-\infty}^{\infty} (W(n) \cos(2\pi(\omega_0 - \omega)n) e^{i\pi D n^2}) \quad (4)$$

$$S_l(\omega) = ae^{i\phi} \sum_{n=-\infty}^{\infty} W(n) n i \sin(2\pi(\omega_0 - \omega)n) e^{i\pi D n^2} \quad (5)$$

Here S_c represents the spectrum of the constant amplitude part and S_l represents the spectrum of the linear amplitude part of the sinusoid.

For the discussion of equations (3-5) we assume the coordinate system of the amplitude and phase spectra to be shifted using the translation $\omega' = \omega - \omega_0$. Accordingly, the frequency origin of ω' is located at the sinusoidal frequency ω_0 . For $D = 0$ the amplitude of the spectra of both parts will be even functions with the spectrum of the second part being 0 at the origin. $S_c(\omega')$ and $S_l(\omega')$ have a local maximum respectively minimum at the origin. The phase of $S_c(\omega')$ is constant with value ϕ within the mainlobe. The phase of $S_l(\omega')$ is odd, it consists of two constant parts (with value $\phi \pm \pi/2$) with a phase jump of π right at the origin. The sum of $S_c(\omega')$ and $S_l(\omega')$ has even amplitude and odd phase ϕ with the value $Ae^{i\phi}$ at the origin. Depending on the ratio of A and a the spectrum may present either a local maximum or minimum at the origin. For all common analysis windows and the sinusoidal model in equation (1) the resulting spectrum has a maximum. As our first result we may conclude that for $D = 0$ the QIFFT estimator provides results that are biased only by the first two sources of bias mentioned above and that the time varying amplitude $a \neq 0$ does not add any additional bias.

For $D \neq 0$ the factor $e^{i\pi Dn^2}$ adds an even phase to the elements of the sum. As a result the magnitude of $S_c(\omega')$ and $S_l(\omega')$ does keep all the characteristics discussed above, notably even symmetry and extreme value characteristics (maximum and minimum). The (unwrapped) phase spectra however are no longer (locally) constant, but both phase spectra have an additional even phase function superimposed. The phase offset of $S_c(\omega')$ does not vanish at the origin and by consequence the phase is biased already for $a = 0$. For $a \neq 0$ the even symmetric phase offset that is applied to $S_l(\omega')$ will destroy the even symmetry of the magnitude of $S(\omega')$ such that the peak maximum moves away from the origin, and therefore, the amplitude and frequency estimates of the QIFFT estimator are no longer correct. Accordingly, the QIFFT estimator suffers from additional bias quite similar as has been shown for the Gaussian window in [7].

3. REDUCING THE BIAS

In the previous section we saw that the source of the bias of the QIFFT estimator is the frequency slope of the sinusoid. A conceptually simple approach to estimate the parameters (A, ϕ, ω) of a sinusoid related to a spectral peak requires two steps:

1. estimate the frequency slope,
2. demodulate the sinusoid and use the QIFFT estimator to find the sinusoidal parameters.

Note, that this approach is in principle equivalent to the MLE for constant amplitude linear FM signals described in [4]. Because the demodulation technique is used for the frequency slope estimation we will first discuss the frequency domain demodulation algorithm. In the following section the frequency slope estimation is described.

3.1. Demodulation

The main objective of the present algorithm is to provide a means to demodulate the sinusoid using only the part of the spectral peak that is accessible for analysis. Because the sinusoidal peak is covered by noise this part will generally be the part of the mainlobe exceeding the noise level. Initially, we assume we are given a frequency slope estimate $\hat{D} = D$ for a peak that is part of a signal spectrum.

In time domain the demodulation can be achieved simply by multiplication with a demodulator signal

$$y(n) = e^{-i\pi \hat{D}n^2}. \quad (6)$$

Multiplication of the signal in equation (6) with the signal equation (1) will remove the frequency slope and keep all other parameters unchanged such that the QIFFT algorithm can be applied. However, the signal we are interested in is observable only via the part of its mainlobe that constitutes the observed spectral peak.

The demodulation algorithm that uses the observed peak to demodulate the sinusoid will be described in the frequency domain using as sources the spectral peak to be analyzed and the spectrum of the deconvolution signal. Assume $S(k)$ is the N -point DFT of the sinusoid to be analyzed and $Y(k)$ the DFT of the demodulator signal. All DFT spectra are calculated such that the origin of the DFT basis functions is in the center of the analysis window. The signal analysis window is $w_s(n)$ and the demodulator signal is windowed using $w_y(n)$. To obtain the demodulated sinusoid

spectrum $X(k)$ we would need to compute the circular convolution

$$X(k) = C \frac{S(k) \circledast Y(k)}{N}, \quad (7)$$

where C is a normalization factor taking into account windowing effects. The demodulator window w_d will be multiplied with the signal window such that the resulting spectrum contains as effective window the product window $w_y(n)w_s(n)$. Therefore, proper normalization would be achieved by means of setting $C = 1/\sum_n(w_y(n)w_s(n))$.

Due to the fact that only part of the sinusoid spectrum is available the normalization factor needs to be adapted. Assume the peak under investigation is denoted by $P(k)$. $P(k)$ is part of the spectrum $S(k)$ and covers B bins. To estimate the impact of the missing part we create a spectral model of the observed sinusoid assuming the initial slope estimate is correct

$$P_m(k) = \sum_n w_s(n) \exp(i\pi \hat{D}n^2) \exp(-\frac{2\pi j}{N}kn), \quad (8)$$

and select a subset $\bar{P}_m(k)$ of B bins around the center frequency $k = 0$.¹ The required normalization factor can now be approximately estimated as

$$C = \frac{1}{\max_k(|\bar{P}_m(k) \circledast Y(k)|)}. \quad (9)$$

Now we can replace $S(k)$ in equation (7) by $P(k)$ and demodulate using the corrected normalization factor C . Some remarks are in order:

- The correction factor will be more precise (lower bias) for demodulator windows that concentrate more energy in the B -bin wide band around frequency 0 of the spectrum. This calls for low side lobes. The demodulator window, however, will as well be applied to the signal. As a result the estimator sensitivity to noise will increase. Accordingly the demodulator window allows to trade-off noise sensitivity and bias. The experimental investigation suggests that the use of the Hanning window as demodulator window w_d is a favorable choice for all analysis windows w_s .
- The compensation of the normalization factor assumes that the amplitude slope $a = 0$ and that the peak model is cut symmetrically with respect to the peak center. To create an optimal correspondance between the compensation factor and the missing part of the signal it is preferable if the spectral peak $P(k)$ that is used for demodulation is as close as possible to the peak model that is used to derive the compensation factor. The comparison of a number of strategies that may be employed to extract the observed peak from the spectrum we found that cutting the peak such that its left and right magnitude have approximately the same value creates the smallest bias. Besides the fact that this method achieves perfect compensation for $a = 0$ there is a second advantage of this method that is related to the impact of the background noise. Assuming the background noise energy to be constant and understanding the maximum border amplitude of the peak as a very rough indicator of the background noise level we may conclude that cutting the peak at its maximum border level could be beneficial because it avoids the parts of the signal where the background noise is dominant.

¹ If B is even the resulting model is not symmetric!

- For parameter estimation from demodulated peaks with the QIFFT estimator it is essential to use the bias correction functions proposed in [8] with correction factors adapted to the effective window $w_y(n)w_s(n)$.

Our experimental investigation shows, that the demodulation kernels $Y(k)$ can be precalculated for a fixed grid of frequency slopes and then linearly interpolated to obtain an approximate demodulation kernel for any given slope. If the length of the analysis windows is M a frequency slope grid with step size $0.025/M^2$ is sufficient to produce estimates that are nearly indistinguishable from the results produced with the non interpolated kernels. To use the complete information that is available in the observed peak we use deconvolution kernels of length $2B + 1$ centered around the maximum of the deconvolution spectrum.

The implementation of the deconvolution can be done in the frequency domain (as described) or in the time domain. Time domain implementation would probably be more efficient if at least the demodulation kernel could be directly stored in the time domain. The possibilities of time domain interpolation of the demodulation kernels have not yet been studied, we believe however, that time domain interpolation would require on the fly generation of the complex kernels from interpolated phase functions. Due to the linearly modulated frequency of the demodulation kernels this will most likely be less efficient than the frequency domain implementation that has been described above.

3.2. Frequency slope estimation

As mentioned above the maximum likelihood (ML) frequency slope estimator for constant amplitude linear FM sinusoids maximizes the amplitude of a demodulated peak [4]. Accordingly the maximization of the amplitude of the demodulated peak using the demodulation algorithm described above can be considered an approximate MLE as long as the amplitude slope is sufficiently small.

To avoid the search of a large grid of frequency slopes we propose to use an approximate initial estimate of the frequency slope \hat{D} , and then to use the frequency slope estimate and two slopes with $\hat{D} \pm D_o$ to create three different demodulations of the observed peak. From the amplitudes of these demodulated peaks a 2nd order polynomial model of the relation between frequency slope and demodulated amplitude can be derived. The maximum of this polynomial is expected to provide a refined estimate of the frequency slope.

The open question we need to address is: how do we get an approximate estimate of the frequency slope? Given the highest order coefficients α_ϕ and α_A of the QIFFT polynomial for amplitude (A) and phase (ϕ) of the peak under investigation the frequency slope estimate for a Gaussian analysis window is [3, 9]

$$\hat{D} = \frac{\alpha_\phi}{\alpha_\phi^2 + \alpha_A^2}. \quad (10)$$

Note the remarkable fact, that the same estimator has been obtained for exponential amplitude evolution in [3] and for a first order approximation of the spectrum of a sinusoid with linear amplitude evolution in [9]. The fact that the amplitude evolution function does not affect the frequency slope estimator leads us to suppose that that equation (10) will provide useful estimates for other windows than the Gaussian window as well. The argument here is that the signal that is obtained after the analysis window has been applied can always be considered to be equivalently generated by means of a Gaussian analysis window and a sinusoid with

appropriately modified amplitude evolution. Because the desired frequency estimate does not change with the amplitude evolution of the sinusoid and because the estimator equation (10) appears to be rather insensitive to small changes of the amplitude evolution of the sinusoid it will be considered as approximate estimator for the frequency slope for arbitrary analysis windows.

The free parameter to select is the frequency slope offset D_o . In general a polynomial approximation improves when the approximation range is decreased. This would call for a small D_o . In the present case, however, the relation between demodulation slope and amplitude of the demodulated peak is covered by measurement noise (due to estimation errors of the amplitude of the demodulated peak, due to the partially observed sinusoidal spectrum, and due to the sampling of the Fourier spectrum by the DFT). The final selection of the D_o parameter will be discussed in section 4.1.

The precision of the frequency slope estimate that is obtained from the maximum of the polynomial is slightly, but consistently improved if the polynomial model is not constructed for the demodulated amplitudes \hat{A}_i but for $\hat{A}_i\sqrt{C_i}$ where C_i is the normalization factor from equation (9). Up to now a theoretical explanation of this experimental finding has not yet been found. Using \sqrt{C} to calculate the demodulated amplitudes will obviously create biased amplitude estimates. For the problem of slope estimation it appears to improve the fit of the polynomial model and therefore, it will be preferred. After the slope has been determined from the maximum of the polynomial a re-normalization can be performed if the amplitude of the supporting points is required.

4. EXPERIMENTAL EVALUATION

The proposed parameter estimation procedure will be evaluated by means of comparing it to the bias correction algorithm proposed in [3] for which Gaussian and Hanning analysis windows are used. The results of that algorithm are denoted as *AS*. Furthermore we use the original version of the demodulation estimator according to [10]. (denoted as *DE*) and the new version that includes the slope enhancement and uses the Hanning window for all demodulation kernels (denoted as *DS*).

The window type that is used will be indicated by adding the letter *G* for Gaussian or *H* for Hanning or *X* for both to the estimator shortcut. The window applied to the demodulation kernels will be equal to the analysis window for *DEX* and Hanning for *DSX*. The Gaussian analysis window is cut such that it has a length of 8σ with σ being the standard deviation of the Gaussian. To facilitate orientation we display the results of the QIFFT estimator as well as the Cramer-Rao bounds for second order polynomial phase estimation with that have been presented in [11]. Note, however, that these bounds have been found for constant amplitude polynomial phase signals, such that they can only be used to provide an approximate idea of the estimator efficiency.

In the experiments we use synthetic test signals with a single sinusoid according to equation (1) with $A = 1$, ω_0 randomly sampled from a uniform distribution over the frequency range $[0.2, 0.3]$, ϕ randomly chosen from a uniform distribution between $[-\pi, \pi]$, and varying slopes a and D . The analysis window covers $M = 1001$ samples in all cases. The frequency slope D is selected from a uniform distribution over interval $[-D_{max}/M^2, D_{max}/M^2]$. Similarly the amplitude slope a is sampled from a uniform distribution over the range $[-a_{max}/M, a_{max}/M]$. The slope ranges are considered realistic for real world signals. Note, that in harmonic signals the frequency slope scales with the partial number

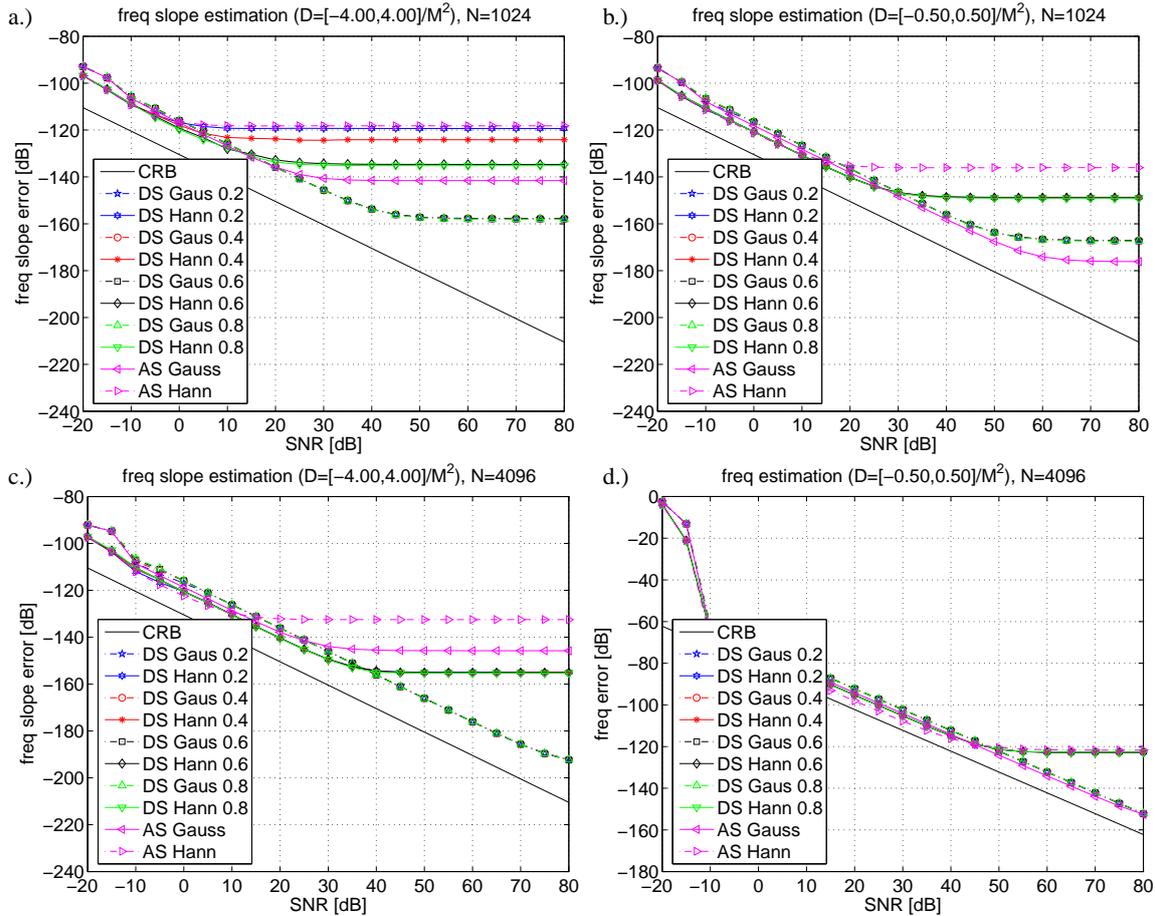


Figure 1: Comparison of the frequency slope estimation errors for the DSX estimator with varying slope offset D_o and the ASX estimator. Window size is $M = 1001$ and sinusoids with strong (a,c) and weak (b,d) amplitude and frequency modulation are considered. DFT size is $N = 4096$ (a,b), and $N = 1024$ (c,d). The CRB for constant amplitude polynomial phase signals is displayed as lower limit. Algorithms using a Gaussian/Hanning window are distinguished by means of solid/dashed lines. See text form more details.

such that for high partials extreme slopes may arise.

Note, that the implementation of the algorithm used for the experimental investigation uses linearly interpolated demodulation kernels as proposed in section 3.1.

4.1. Frequency slope estimation

In the first experiment we investigate into the frequency slope estimation. In Figure 1 we compare the enhanced demodulator *DSX* with the *ASX* method according to equation (10). Because the *DEX* estimator uses the frequency slope estimate provided by *ASX* directly we don't consider *DEX* here. We use two different zero padding factors (FFT size $N = 1024$ and $N = 4096$) and two different sets of modulation ranges, the strong modulation is using $D_{max} = 4$ and $a_{max} = 1$, while for weak modulation we select $D_{max} = 0.5$ and $a_{max} = 0.15$. Note, that the weak modulation range approximately covers the interval for that the *ASH* bias correction has been derived in [3]. The *DSX* estimator is operated with a set of demodulation offsets $D_o \in [0.2, 0.4, 0.6, 0.8]/M^2$.

The results of the experiment are shown in Figure 1. There are a number of conclusions that can be drawn from these figures. First, we find that for strong modulation the *DSX* method has significantly lower bias than the *ASX* method. Second, we observe that for the Hanning window the *DSH* estimator achieves a reduction of the estimation bias by 2 – 30dB. The smallest improvement is achieved for weak modulation and large oversampling factor. The only case where the *ASX* estimator significantly outperforms *DSX* is weak modulation with small oversampling factor and Gaussian analysis window. This could have been expected because the *ASG* estimator is exact for the Gaussian analysis window and the small oversampling factor does not influence this estimator. As expected the Hanning window has larger bias than the Gaussian window but at the same time it is less sensitive to noise by about 4dB. In general the *DSX* are more sensitive to noise by about 2 – 3db.

Considering the demodulation offset D_o we find that the offset has a significant impact only for strong modulation with small oversampling factor and Hanning window. This is related to the

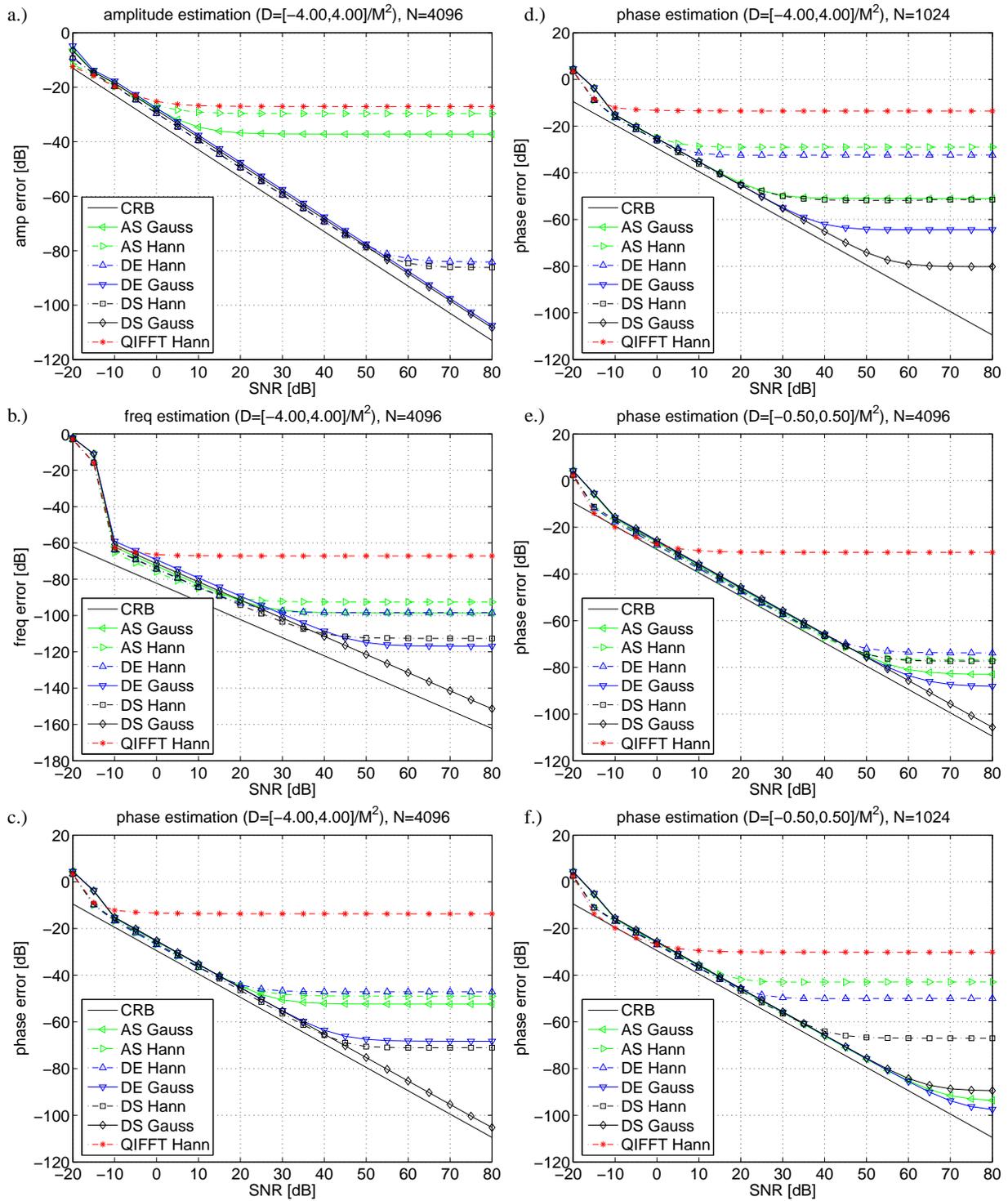


Figure 2: Comparison of the estimation errors for the different parameter estimators using window size $M = 1001$ and FFT size $N = 4096$ and (strong) linear AM/FM with $D_{max} = 4$ and $a_{max} = 1$ (a-c). Figures (d-f) show phase estimation errors for different modulation limits and FFT sizes. The CRB for constant amplitude polynomial phase signals is displayed as lower limit. Algorithms using a Gaussian/Hanning window are distinguished by means of solid/dashed lines. See text form more details.

fact that the initial frequency slope estimate of the ASH that is the basis of the slope refinement in DSX is rather bad, such that the model needs to compensate a larger range of slope errors. Moreover the amplitude estimation is less precise for smaller oversampling factors such that a larger slope offset may be required to obtain a polynomial model that captures the underlying relations. For $D_o = 0.5$ we get nearly optimal results for all cases which is why we select this value for the following experiments.

4.2. Bias correction

After having discussed the properties of the frequency slope estimation we now investigate into the main topic of this paper, the bias reduction. Due to space constraints we will only discuss a few of the experiments we have conducted. We will discuss the results for all parameters for strong modulation with $D_{max} = 4/M^2$ and $a_{max} = 1/M$ and an FFT size of $N = 4096$. Furthermore we select the phase bias reduction as an example and discuss the bias reduction for the phase estimate for weak and strong modulation and FFT sizes $N = 1024$ and $N = 4096$.

The results of the bias reduction for strong modulation and $N = 4096$ are displayed in the left column of Figure 2. As expected the amplitude estimate a.) of ASX is strongly biased due to the fact that the amplitude trajectory model does not match the signal. DEX and DSX are both similar and better than ASX. Note, that the improved frequency slope estimate of DSX hardly improves the amplitude estimate compared to DEX and that the increase of the noise sensitivity of DEX and DSX is negligible. For frequency b.) and phase estimation c.) DSX has by far the smallest bias (compared to the other estimators using the same analysis window). DEH and ASH perform approximately similar for both for frequency and phase estimation. Given that DEX and ASX estimators both use the same frequency slope estimate this shows that the bias of these two estimators is due to the error in the frequency slope estimate which is improved by the refined slope estimate of DSX.

The increase of the noise sensitivity for the demodulation algorithms is negligible for phase estimation. For the frequency estimator the use of the Hanning window instead of the analysis window is clearly diminishing the noise sensitivity when the analysis window is Gaussian.

The right column of Figure 2 shows the phase bias removal for all the experimental settings that were used in the evaluation of the frequency slope estimation. A close inspection of the results reveals that the performance of the bias removal is directly related to the performance of the frequency slope estimation. This can be expected because any error in the frequency slope estimate will translate into an error in the bias correction algorithm.

As a summary of the experimental investigation of the algorithm using synthetic signals we conclude that compared to the QIFFT estimator all the bias reduction algorithms dramatically reduce the estimation bias. Compared to the recent ASX estimator the simple and enhanced demodulation algorithm both provide a significant reduction of the estimation bias especially if the range of the modulation is not confined to the rather limited range of values that has been considered in [3]. Comparing the DEX and DSX algorithms we have shown that the enhanced slope estimation has a direct and significant impact on the bias of the sinusoidal parameters. Due to the fact that the frequency slope bias of the DEX algorithm increases with the modulation we expect that the DSX

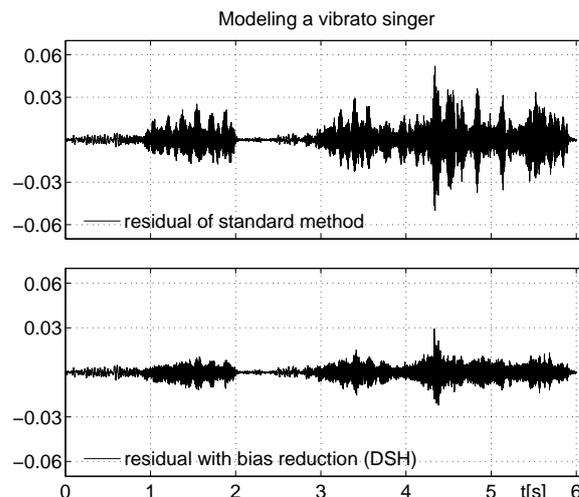


Figure 3: Residual signal of a vibrato tenor singer using QIFFT estimator (top) and the enhanced demodulation method DSH (bottom).

estimator is especially advantageous if the modulation is strong. The possibility to freely select the demodulator window improves the noise sensitivity in case the Gaussian window is used as analysis window.

4.3. A real world example

To demonstrate that the advantages of the proposed estimator are effective in real world situations we have implemented the bias reduction methods in a complete additive modeling system. The theoretical investigation has been restricted to cover the case of resolved sinusoids, only. For real world applications, however, the algorithm has to prove that it will act gracefully when the underlying model no longer holds (transients, unresolved sinusoids due to reverberation, ...). The major problem in real world signals is related to the fact that the enhanced frequency slope estimation described in 3.2 may produce extreme values whenever the underlying signal model does not match the observed peak. In these cases the method may for example try to model the transient or nearby sinusoids by means of extreme slopes.

To prevent the degeneration of the estimator we use a number of tests that are designed to allow us to detect the cases for that the signal model that is used to analyze the peak does not hold. The tests that verify the reliability of the second order polynomial model of the relation between demodulation slope and amplitude are: verification that the extremum of the polynomial model is a local maximum, verification that the amplitude that is obtained with the optimal demodulation slope is larger than the amplitude obtained with the initial slope estimate, verification of that the slope offset to reach the optimal slope is within $\pm 2D_o$. If one of these tests fails the polynomial representation of the slope and amplitude relation is considered unreliable and the DEX estimator is used as a fallback.

The test that verifies the validity of the linear AM/FM sinusoidal representation is based on the center of gravity of the energy (the mean time) of the signal related to the spectral peak under investigation. If the mean time is larger than the maximum mean time that can be expected for the signal model equation (1) then

we can assume that the peak is related to a sinusoid with transient amplitude evolution [12]. In this situation the exponential amplitude evolution used by the ASX estimator is more appropriate than the linear AM and therefore the ASX estimator is used. Note, that the ASX and DEX estimators are sub modules that are required for the DSX estimator anyway such that the fallback solutions do not require additional costs in terms of implementation or calculations.

freq band	ASH	DEH	DSH
full	-4.19	-4.72	-5.04
0-2kHz	-3.13	-3.75	-4.05
2-4kHz	-7.32	-8.40	-9.33
4-6kHz	-5.78	-6.90	-7.32

Table 1: The reduction of the energy of the residual signal obtained with the different bias reduction algorithms. The performance of the algorithms varies with the frequency band.

For the last experiment we compare the estimators by means of the energy of the residual signal of an harmonic model of a tenor singer. The signal contains strong vibrato, and therefore, the bias due to the non-stationary parameters is expected to be significant. The harmonic models contain a maximum of 30 sinusoids at each time instant. We calculate the variance of the residual signal for the QIFFT, DEH, DSH, and ASH methods for a signal window of 800 samples and a FFT size of 4096 samples. The variance of the residual signal is compared to the QIFFT estimator and the reduction of the residual energy in different frequency bands that can be achieved with each estimator is listed in table (1).

From table (1) we can conclude that all bias reduction methods achieve significant improvements of the residual energy. It is interesting to compare the performance in the different frequency bands. In the low band the improvement is in the range from 3-4dB. The improvement is less pronounced because the FM modulation extend is low. In the mid band range the FM modulation becomes stronger and the reduction methods achieve residual energy reduction from 7.3-9.3dB. For the highest band the FM modulation is still stronger, but the noise level is higher as well such that the reduction of the residual energy is not as strong.

The advantage of the demodulation methods over ASH is clearly visible. The DEX estimator improves the reduction of the ASH estimator by 0.5-1.2dB. The DSX estimator is clearly the best with an improvement compared to the ASH estimator by 0.8-2dB. The residual signals for the QIFFT and DSH estimator are shown in figure Figure 3. The reduction of the residual is clearly visible.

5. CONCLUSIONS

In the present paper we have shown that an efficient bias reduction strategy for estimation of sinusoidal parameters consists of a frequency slope estimation and demodulation prior to application of the standard QIFFT estimator. The procedure significantly reduces the bias of the standard estimator. It does not require the use of a Gaussian analysis window and does work for a much larger range of modulation depths than a recently proposed algorithm. The computational costs are significantly higher than those for the standard estimator (\approx factor 8). However, they are sufficiently low such that real time estimation of some tenth of sinusoids from audio signals can be achieved. By means of investigation into the reduction of the residual energy that can be obtained for a real world

vibrato signal we have shown that the proposed enhanced demodulation estimator is effectively working in real world situations. It has been shown that compared to the standard QIFFT estimator the reduction of the residual error depends on the frequency range and can be as large as 6-9dB.

6. REFERENCES

- [1] X. Amatriain, J. Bonada, A. Loscos, and X. Serra, "Spectral processing," in *Digital Audio Effects*, U. Zölzer, Ed., chapter 10, pp. 373-438. John Wiley & Sons, 2002.
- [2] T. F. Quatieri and R. J. McAulay, "Speech transformation based on a sinusoidal representation," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 34, no. 6, pp. 1449-1464, 1986.
- [3] M. Abe and J. O. Smith, "AM/FM rate estimation for time-varying sinusoidal modeling," in *Proc. Int. Conf. on Acoustics, Speech and Signal Processing*, 2005, pp. 201-204 (Vol. III).
- [4] T. Abatzoglou, "Fast maximum likelihood joint estimation of frequency and frequency rate," in *ICASSP*, 1986, pp. 1409-1412 VOL. II.
- [5] S. Saha and S. M. Kay, "Maximum likelihood parameter estimation of superimposed chirps using monte carlo importance sampling," *IEEE Trans on Signal Proc.*, vol. 50, no. 2, pp. pp. 224-230, 2002.
- [6] J. S. Marques and L. B. Almeida, "A background for sinusoid based representation of voiced speech," in *Proc. Int. Conf. on Acoustics, Speech and Signal Processing*, 1986, pp. 1233-1236.
- [7] G. Peeters and X. Rodet, "SINOLA: A new analysis/synthesis method using spectrum peak shape distortion, phase and reassigned spectrum," in *Proc. Int. Computer Music Conference*, 1999, pp. 153-156.
- [8] M. Abe and J. O. Smith, "Design criteria for the quadratically interpolated FFT method (I): Bias due to interpolation," Tech. Rep. STAN-M-117, Stanford University, Department of Music, 2004, available at <http://ccrma.stanford.edu/STANM/stanms/stanm114/index.html>.
- [9] G. Peeters, *Modèles et modification du signal sonore adapté à ses caractéristiques locales*, Ph.D. thesis, Université Paris 6, 2001, available at http://recherche.ircam.fr/equipes/analyse-synthese/peeters/ARTICLES/Peeters_2001_PhDThesisv1.1.pdf, french only.
- [10] A. Röbel, "Estimation of partial parameters for non stationary sinusoids," in *Proc. Int. Computer Music Conference (ICMC)*, 2006.
- [11] B. Ristic and B. Boashash, "Comments on "The Cramer-Rao lower bounds for signals with constant amplitude and polynomial phase"," *IEEE Transactions on Signal Processing*, vol. 46, no. 6, pp. 1708-1709, 1998.
- [12] A. Röbel, "A new approach to transient processing in the phase vocoder," in *Proc. of the 6th Int. Conf. on Digital Audio Effects (DAFx03)*, 2003, pp. 344-349.